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# Supplier Selection via Tournaments

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In this paper, we study the performance of a sourcing mechanism gaining popularity in industrial procurement environments; a *tournament*. Under a tournament, a buyer initially procures her parts from two suppliers with possibly different quality levels, for $T$ time periods, i.e., she parallel sources. During this time, the buyer is able to observe noisy signals about the suppliers' quality. At time $T$, she selects the supplier with the highest observed performance and awards it the remainder of her business. We characterize the optimal duration of the tournament as a function of various market parameters, including information and investment costs. Furthermore, we demonstrate that a tournament can be more profitable for the buyer than selecting the highest quality supplier at time $T = 0$ and sole sourcing entirely.

*Key words*: parallel or dual sourcing; procurement; quality; asymmetric information; investment
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## 1. Introduction

As a growing number of companies are increasing their reliance on outside suppliers for integral inputs into the production process, the *supplier selection mechanisms* used to select these critical suppliers is increasingly important. Particularly in high-tech and manufacturing industries such as electronics, aerospace, and automotive, a supplier can greatly influence the final quality of a buyer's product, via the supplier's technology capabilities, skilled worker base, reliability of delivery, and product reliability.[2] (Fn2) While there are varying opinions as to the relative importance and effectiveness of different types of quality control measures, corporate and societal cultures (Yeung et al. 2005) in achieving high quality products, there is no disputing that the quality of suppliers, their willingness to invest in new technology, and flexibility in responding to the marketplace and buyer's demands are critical for the final success of a buyer-supplier relationship.

While the importance of a supplier's quality, e.g., leadtime, during the supplier selection process is a widely acknowledged and studied area in operations management (e.g., Yan et al. 2003; Tempelmeier 2002), this literature generally assumes that the supplier's quality level is fixed and exogenously determined and is known to both the buyer and supplier; under this setting, the papers study how to optimally purchase from the set of existing suppliers. That is, there is no asymmetry in information between the buyer and suppliers concerning their quality levels. In contrast, in high-tech and manufacturing setting where quality is of pivot importance, a supplier's quality level is often unknown (or imperfectly known) to the buyer and suppliers. Furthermore, a supplier is often able to improve his quality by making investments in the relationship, for example, by training his employees or investing in new and better equipment. While the buyer would like the supplier to undertake costly investments in the relationship so as to improve his quality, the buyer is rarely willing to pay the sup-

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[2] One need only point to the unfortunate events between Ford Motor Company and Firestone Tire company to understand the critical importance of these quality attributes in a buyer-supplier relationship (Stimson et al. 2000).

Fn3 plier for these investments.[3] Ideally, the buyer would be able to perfectly assess the suppliers' quality and select the optimal investment levels for the suppliers to undertake. However, buyers who do not "foot the bill" for desired investments are rarely able to dictate and perfectly observe the suppliers' investments, i.e., the suppliers' investments are unobservable and unverifiable. This asymmetry in information implies that a buyer who faces a set of suppliers of unknown quality and unobservable investments must design a sourcing arrangement that allows her to both extract some information about the suppliers and encourage (costly) investments on their part.

One method used by the Department of Defense,
Fn4 Solectron, and Toyota[4] (amongst others) to identify high quality suppliers and provide them with the incentive to improve their quality is to initially dual (parallel) source with (possibly new or incumbent) suppliers. As the buyer observes the suppliers' quality (via, for example, his output), the buyer begins to redistribute her business among the suppliers by increasing her reliance on observed high quality suppliers. The buyer may choose to terminate her business with one (or more) observed low quality suppliers. By awarding greater portions of her business to a well-performing supplier, the buyer creates a tournament in which the suppliers compete for the "prize"—a larger share of the buyer's business. This type of sourcing arrangement can be a tremendous success for the buyer. In the 1970's, the Department of Defense found itself saddled with a highly defective/low quality F100 engine produced by Pratt and Whitney. As the sole producers of this engine, Pratt and Whitney were unresponsive to the Air Forces pleas for a higher quality product. In order to combat Pratt and Whitney's complacency, the government decided to outsource a portion of its engine business to General Electric. The government made each company's share of future business contingent on its products' quality and performance. The resulting increase in quality due to competition was tremendous. In testimony before the House Appropriations Subcommittee on Defense in 1979, General Lew Allen, Air Force Chief of Staff, explained: "We are concerned about the motivation and incentive of Pratt and Whitney to correct this engine. . . . [The] best way to insure that we were adequately addressing the problem was to generate some competition. . . . The approach with General Electric . . . is an attempt . . . to develop a true competitive situation within the engine industry" (Drewes 1987, p. 145).

In this paper, we consider a stylized model of this supplier selection mechanism and evaluate his optimal structure and performance. We consider a buyer who faces two types of suppliers with possibly different quality levels. The buyer would like to procure her parts from the highest final quality supplier, where final quality is determined by a supplier's current quality and any costly investment undertaken as a result of the buyer-supplier relationship. The buyer is considering using a tournament of duration $T$; namely, the buyer will parallel source from both suppliers and observe signals of their quality level for $T$ time periods and then award a sole sourcing contract for the remainder of her demand to the supplier who exhibits the highest quality level.

As independent entities, both the buyer and suppliers will behave strategically and act in their own best interest. Acknowledging this, a buyer must take into account the supplier's private information and strategic behavior when selecting and designing a supplier selection mechanism. Using game theory, the three main questions this paper seeks to answer are:

- What is the relationship between suppliers' investment levels and $T$?
- How do the information in the marketplace and cost structure of investment affect the optimal $T$?
- How do the information in the marketplace and cost structure of investment influence the effectiveness of a tournament?

Via numerical analysis, we compute the optimal duration of the parallel sourcing period as a function of the supplier characteristics. We are able to provide insights into how the suppliers' optimal investment levels are affected by the choice of $T$, the suppliers' qualities, cost of investment, and the "noisiness" of the quality signal, as well as the probability that the buyer incorrectly selects the low quality supplier at time $T$.

We find that the benefits of a tournament during the selection process are two-pronged. As stated before, the buyer often does not know the suppliers' qualities, hence parallel sourcing gives the buyer a chance to observe and learn more about the suppliers' qualities. In addition, parallel sourcing improves the buyer's profitability via heightened competition among suppliers. Interestingly, we find that even if the buyer were able to perfectly assess the suppliers' quality and choose the highest one, she may still prefer to parallel source from both suppliers for some portion of time. This is due to the "tournament" structure of the procurement mechanisms and the positive effect that increased competition has on the supplier's willingness to invest. Furthermore, these results hold whether the suppliers' investment decisions are fixed (one-time irreversible decision) or variable (the supplier can se-

[3] For example, buyers are often worried that supplier-improving investments will be shared with other customers, and therefore are reluctant to pay for them (Caltabiano 2001; Wong 2001).

[4] See Richardson and Roumasset (1995) and Treece and Rechtin (1997) for a brief discussion of Toyota's parallel sourcing experience.

lect his investment level at more than one point in time).

The organization of the rest of our paper is as follows. In Section 2, we provide a description of our problem. We review the relevant literature on tournaments and competition in procurement mechanisms in Section 3. In Sections 4 to 6, we examine the equilibrium of the tournament game under various market settings and demonstrate several structural properties of the equilibrium strategies through numerical simulations. We conclude in Section 7 with managerial insights.

## 2. Model

We focus on a buyer who wishes to purchase an input, $a$, for the production of her final product, $A$, over a finite time horizon [0, 1], where product $A$ faces a demand rate of $2D$ units per period. The buyer faces two types of potential suppliers who have identical constant marginal production cost, $c$. A supplier can either be of low or high quality, $q_L$ and $q_H$, where $q_L > q_H$. Without loss of generality, we scale $q_L$ and $q_H$ to be numbers between 0 and 1. We can interpret $q_i$ ($i = L, H$) as the probability that a unit of $a$ produced by supplier type $i$ ($i = L, H$) is defective.

The buyer sells each unit of her final product $A$ at a market price $p^M$, and offers the supplier(s) a per unit price of $p^S$ for each unit of input $a$, where $p^S < p^M$. We assume that a supplier that enters into a relationship with the buyer can improve his quality level by undertaking a costly investment $I_i$ ($i = L, H$). That is, a supplier's final quality is a function of his inherent quality level, $q_i$, and the investment he undertakes to improve his quality. While a supplier bears the entire cost of investment, the improvement in quality benefits both the supplier and buyer through higher (expected) defect-free output levels. The cost of investment $I$ is given by a real function $C(\cdot)$ which satisfies the following conditions.

Assumption 1. *$C : [0, U) \rightarrow R^1_+$ is twice continuously differentiable and strictly increasing with $C'(0) = 0$, $\lim_{x\rightarrow U} C'(x) = +\infty$, and $C'' > 0$ for all $x \geq 0$ (namely, $C$ is strictly convex).*

While the buyer faces a demand of $2D$ per period for product $A$, the number of units $A$ that result in a final sale depends on a supplier's quality. Customers return any defective units of product $A$ back to the buyer for a full refund and are then a lost sale; the probability that a customer will return the final product for a full refund is increasing in $q$. We assume that the buyer cannot accurately infer a supplier's quality from the number of returned units, i.e., the number of returned units is a noisy signal of supplier quality. This is because there is a positive probability that other components or external factors in the buyer's supply chain render the final product defective. We assume that the buyer is unable to detect (or it is cost prohibitive for the buyer to detect) which component causes the final product to be defective; the buyer is only able to observe the number of units returned by customers; hence, a suppliers' final quality is *unverifiable*. We capture these external factors by a random factor $\epsilon$. In particular, we model $\epsilon$ as the time-average fraction of units supplied by supplier $i$ per unit time that result in a final sale over some time interval [0, $t$], denoted by $F(q_i, I_i)$ ($i = L, H$). $F(q_i, I_i)$ is assumed to be a truncated normal random variable with a mean of $f(q_i, I_i)$ and a variance of $\sigma^2$. That is, the number of final units of $A$ that result in final sales (are not returned) is determined by the supplier's quality, investment, and a complex interaction of factors in the supply chain that is captured by $\sigma$ (that are not attributable to the supplier's quality). From the definition of $F(q_i, I_i)$, it is reasonable to expect that the variance $\sigma^2$ decreases in $t$. We assume that $f(q, I)$ and $\sigma$ have following properties.

Assumption 2. *$\sigma$ is a continuously decreasing function of $t$.*

Assumption 3. *$f(q, I) : [0, 1] \times [0, U) \rightarrow R^1_+$ is continuously differentiable in $q$ and twice continuously differentiable in $I$. Moreover, $f(q, I)$ is strictly decreasing in $q$, strictly increasing in $I$, strictly concave in $I$, and $(\partial/\partial I)f(q, I)|_{I=0} > 0$.*

While the buyer cannot verify a supplier's quality in our model, she can observe noisy signals about quality. Therefore, she must confine herself to selection mechanisms that only require knowledge about a supplier's observed quality. The buyer uses a tournament to select a supplier. Initially, she splits her demand equally and parallel sources with both suppliers for some initial duration $T$, i.e., each supplier is offered a contract ($p^S$, $D$, $T$). At time $T$, she switches over her entire order to the supplier whose store resulted in the highest observed final sales, and offers that supplier the contract ($p^S$, $2D$, $1 - T$). $T$ is assumed to take on a set of discrete values $\{t_1, t_2, \ldots, t_k\}$ in interval [0, 1]. We assume that suppliers are paid for each unit that results in a final sale, returned merchandize results in lost sales, and defective units have a salvage value of zero.[5] Fn5

In Section 4, we initially analyze the base case in which ($i$) there is exactly one high quality supplier $q_H$ and one low quality supplier $q_L$, ($ii$) the buyer does not know which supplier is of high quality, and ($iii$) suppliers can make a one-time irreversible investment $I_i$ at $T = 0$. We model the tournament as a two-stage game

[5] We ignore all holding and set-up costs and assume that the production of $a$ and demand realization for $A$ are instantaneous.

and numerically solve for its equilibrium under various market settings. We then consider extensions to our model in Section 6, by relaxing assumptions (*i*) to (*iii*).

## 3. Tournaments in Practice and Theory

Tournaments have been widely used by marketing departments as a means to motivate workers to exert costly effort and increase sales. In sales tournaments, the employee(s) with the highest sales are awarded more lucrative accounts and/or given bonuses. These tournaments are generally "relative" performance mechanisms, where the winner is the employee with the highest observed output. Sales tournaments have been extensively studied in the marketing and economics literature (Lazear and Rosen 1981; Green and Stokey 1983; Kalra and Shi 2001). A strong assumption made in all of these papers (and relaxed in this one) is that agents are *ex ante* identical. This assumption implies that the principal does not need to concern herself with selecting the "wrong" agents, but rather with how to design the prizes so as to attain the maximum output/highest innovation value from her workers. More recently, Cachon and Lariviere (1999), motivated by General Motors method for allocating the lucrative cars to competing dealers, study the performance of a "turn-and-earn" reward mechanism in a two period model. Under a "turn-and-earn" mechanism, a dealer's future allocation of resources is determined by its past sales. While the focus of their analysis is on the mechanism's ability to coordinate the supply chain, they also assume that dealers are *ex ante* identical; hence, again obviating the need to select the "best" or highest quality one.

Similarly, tournaments have been used repeatedly by R&D and product development departments in various industries. Fullerton and McAfee (1999) describe the use of a research tournament in 1829 to select an engine for the first-ever passenger line between two British cities, and in recent years to develop America's most energy-efficient refrigerator. Toyota has also applied the principle of tournaments in their product development process. Referred to as set-based concurrent engineering, Toyota routinely develops and tests multiple product types simultaneously, with the "best" one being selected after observing the
Fn6 various options' performance (Sobek et al. 1999).[6]

In addition, tournaments have also been used during the supplier selection process by the Department of Defense (Drewes 1987), as well as by electronic manufacturers (Caltabiano 2001). In these settings, the buyer awards her future business based on the past performance of suppliers. The closest paper to our own that addresses the use of tournaments in supplier selection is by Seshadri (1995). Seshadri considers an alternative tournament arrangement where several suppliers bid to be one of two suppliers chosen to produce a fixed (equal) quantity. After suppliers bid and the two lowest bidders are chosen for production, the suppliers can choose to exert effort to reduce their total cost of production. After production, the suppliers will be paid their actual cost plus a portion of a divisible prize, with the lower cost supplier being awarded a larger share of the prize. Seshadri finds that, while under dual sourcing, by definition, two suppliers of unequal costs must be paid instead of one, the presence of competition and a prize lessens the cost of effort. At times, the presence of competition and associated reduction in cost is greater than the increase in having two suppliers and a dual source arrangement is optimal. A key assumption in Seshadri's paper is that the buyer can observe a supplier's actual costs and hence accurately infer and reward effort.

To the best of our knowledge, tournaments as a supplier selection mechanism in the face of unknown supplier quality and unverifiable investment have never been studied in the literature, despite their use in practice. The bulk of the mechanism design/game theoretic literature in supplier selection mechanisms focus on the use of auctions and cost-plus contracts (please see Elmaghraby 2000 for a review of this literature). Appropriately designed auctions have been shown to be the optimal procurement mechanism in a variety of settings when a supplier's quality is unknown but observable/verifiable (e.g., Dasgupta and Spulber 1989). Key to the credibility and effectiveness of the auction is the assumption that the buyer can write and award contracts that are enforceable in a court of law. For example, if the supplier's quality in our model were verifiable, i.e., $\sigma = 0$, or the buyer could cost-effectively identify the cause of a defect, then the buyer could award a contract that specifies the number of non-defective items to be procured from a supplier. However, if the supplier's investment is intangible or of nonmonetary form (e.g., human capital), or if the benefits of the investment are idiosyncratic, then a supplier's final quality is not verifiable and an auction of contracts cannot be used. In our model, this implies that the buyer is unable to write and auction contracts with suppliers that are based on the number of units that result in a final sale.

## 4. Supplier Selection via a Tournament

During the first $T$ periods of the tournament, the buyer can observe noisy signals about the suppliers' types by

[6] Pich et al. (2002) refer to this process as selectionism.

observing the number of units returned. If $T$ is very small, then it is possible for the buyer to incorrectly choose the low quality supplier at time $T$, due to the noisiness of the quality signal. However, a small $T$ implies a larger prize (sole source contract over time horizon $1 - T$), which may stimulate higher investment levels from the suppliers. On the other hand, as the duration of parallel sourcing increases, the buyer is able to gather more information about each supplier and to make a more informed decision at $T$, but the size of the prize decreases. $T$ plays two roles in this setting: (*i*) a determinant of the number of "samples" used to decide which supplier to select, and (*ii*) a motivator for quality-improving investment. The first role is a familiar one from statistical sampling (i.e., the Law of Large Numbers), the second highlights the strategic nature of the selection process, and introduces the need for game theoretic analysis.

The determination of the optimal contract duration $T$ can be solved as a Stackelberg game. The buyer presents both suppliers with the same contract ($p^S$, $D$, $T$), that satisfies each supplier's individual rationality constraint. Without loss of generality, we assume that the suppliers have an outside opportunity cost of zero and are therefore willing to accept any contract that yields them a nonnegative expected profit. After both suppliers accept the contract, they both simultaneously decide on and undertake their optimal investment levels.

We first consider the setting where the buyer and suppliers know that there is exactly one high quality and one low quality supplier in the market. The high quality supplier who is presented with a contract of the form ($p^S$, $D$, $T$) will solve the following profit maximization problem:

$$\max_{I_H \in [0,U)} DT(p^S f(q_H, I_H) - c) + 2D(1-T)\Pr(F_H \ge F_L) \times (p^S f(q_H, I_H) - c) - C(I_H) \quad (1)$$

where $\Pr(F_H \ge F_L) \equiv \Pr(F(q_H, I_H) \ge F(q_L, I_L))$ is the probability of high quality supplier winning the tournament and $I_L$ is the low quality supplier investment level. Similarly, the low quality supplier will solve the following optimization problem, given an investment level of $I_H$ from the high quality supplier:

$$\max_{I_L \in [0,U)} DT(p^S f(q_L, I_L) - c) + 2D(1-T)\Pr(F_L > F_H) \times (p^S f(q_L, I_L) - c) - C(I_L) \quad (2)$$

where $\Pr(F_L > F_H) \equiv \Pr(F(q_L, I_L) > F(q_H, I_H))$.

The objective function of supplier $q_H$ in (1) is continuous in $I_H$ in interval $[0, U)$ and approaches $-\infty$ as $I_H$ tends to $U$ from left for all values of $I_L$, $T$, and other parameters. So (1) attains its maximum inside interval $[0, U)$. We assume that the global maximum of the objective function (1) is unique for every possible $(I_L, T) \in [0, U) \times \hat{T}$ with other parameters given.

ASSUMPTION 4. *Given model parameters ($q_H$, $q_L$, $\sigma$, $D$, $p^M$, $p^S$, $c$), the respective global maximums of* (1) *and* (2) *are unique for every* $(I_L, T)$ *and* $(I_H, T)$ *in* $[0, U) \times \hat{T}$.

This is a relatively mild assumption since it would be satisfied under common assumptions such as concavity. For instance, sufficient conditions for it to hold include (1) and (2) being strictly concave or strictly unimodal in $I_H$ and $I_L$, respectively.

Under the above assumptions, pure strategy Nash equilibrium investment levels $I_H^*$ and $I_L^*$ exist for every $T \in \hat{T}$.

PROPOSITION 1. *Suppose Assumptions* 1 *to* 4 *hold. Given a contract ($p^S$, $D$, $T$) offered by the buyer, there exist supplier pure strategy Nash equilibrium investment levels* $(I_H^*, I_L^*)$.

The proof is provided in Appendix A. Anticipating the equilibrium investment levels $(I_H^*, I_L^*)$ exerted by the suppliers under ($p^S$, $D$, $T$), the buyer chooses the duration of the contract to maximize her profit[7]: Fn7

$$\max_{T \in \hat{T}} (p^M - p^S)\{DT(f(q_H, I_H^*) + f(q_L, I_L^*)) + 2D(1-T) \times (\Pr(F_H^* \ge F_L^*) f(q_H, I_H^*) + (1 - \Pr(F_H^* \ge F_L^*)) f(q_L, I_L^*))\} \quad (3)$$

where $\hat{T} \equiv \{t_1, t_2, \ldots, t_k\} \subset [0, 1]$.

Moreover, the objective function of the buyer as given in (3) takes on a finite number of values and attains its maximum value at the corresponding optimal parallel sourcing duration $T^* \in \hat{T}$. As a result, $\{T^*, (I_H^*, I_L^*)\}$ is a pure strategy subgame perfect Nash equilibrium (SPNE) of this buyer-supplier game. We summarize it as Proposition 2.

PROPOSITION 2. *Given Assumptions* 1 *to* 4 *hold, there exists a pure strategy subgame perfect Nash equilibrium* $\{T^*, (I_H^*, I_L^*)\}$ *to the Stackelberg game described in this section.*

In principle, one can resort to envelope theorems to investigate the dependence between equilibrium strategies $\{T^*, (I_H^*, I_L^*)\}$ and model parameters ($q_H$, $q_L$, $\sigma$, $D$, $p^M$, $p^S$, $c$). However, the equations obtained by applying envelope theorems to the objective functions of the buyer and suppliers arising from our Stackelberg game model are neither insightful nor tractable. As a result, we illustrate the structural properties of the

[7] If the suppliers do not make an investment decision, then the buyer's problem reduces to selecting the $T$ that optimally trades-off the probability of selecting the high type at time $T$ with the cost of parallel sourcing from the low type during $[0, T]$.

**Table 1 Parameter Values Used During Simulations**

| Parameter | Possible Values |
|---|---|
| $q_H$ | 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4 |
| $q_L$ | 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.6 |
| $\sigma$ | 0.1, 0.5, 1, 3, 5 |
| $\chi$ | 1, 5, 10, 20, 30, 50, 60, 70, 75, 100 |
| $p^S$ | 1.5, 2, 2.5, 3, 3.5, 4 |
| $D$ | 10, 25, 50, 100, 150, 200, 1500, 2000 |

equilibrium strategies through numerical simulation in Section 5.

## 5. Numerical Results

Using numerical computation, we were able to derive some interesting observations with regards to the optimal contract duration for the buyer as well as the effects of a tournament on the buyer's profit and suppliers' quality improving investments. In our experiments, we assumed that

$$C(I) = \chi\left(\frac{U^2}{U - I} - (I + U)\right)$$

$$f(q, I) = 1 - q^{(1+I)} \tag{4}$$

with constants $U > 0$ and $\chi > 0$. The functional forms of $C(I)$ and $f(q, I)$ in (4) are examples for which Assumptions 1 and 3 hold true.[8,9] One remark on the two forms of $f(q, I)$ mentioned above and in Footnote 8 is that the marginal increase in the average number of delivered non-defective components due to additional investment in quality improving effort is higher for the high quality supplier than that for the low quality supplier. In the computations, we set $U = 2.5$, $p^M = 5$ and $c = 1$. In each simulation, we drew the model parameters from the possible values listed in Table 1. Possible contract durations are $T \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$.

### 5.1. Optimal Tournament Duration and Equilibrium Investment Levels

As a starting point, it is quite revealing to examine how the suppliers' optimal investment levels are affected by $T$, $q_H$, and $q_L$. Consider the two panels in Fig. 1. The left panel in Fig. 1 plots the buyer's expected profit $\pi_{\text{Buyer}}$, $I_H^*$, $I_L^*$, and $\Pr(F_H > F_L)$ when $q_H = 0.2$, $q_L = 0.45$, $\sigma = 0.1$, $\chi = 75$, $p^S = 4$, and $D = 100$; while the right panel does so for $q_L = 0.23$, with all else held equal. Both panels show the optimal investment levels for suppliers $q_H$ and $q_L$ for different contract durations, as well as the buyer's expected profit as a result of those investment levels. Furthermore, the figures plot the probability that the buyer selected the high quality supplier at time $T$.

Figure 1 demonstrates some interesting optimal investment behaviors. There are two factors that influence $q_H$'s investment level; the size of the "prize" and the probability that $q_L$ is selected at time $T$. The buyer would like to set $T$ so as to elicit the largest possible investment from $q_H$. When the quality differential $q_L - q_H$ is small, $q_H$ invests at high levels as $T$ increases, since the probability that $q_L$ is selected with only a short parallel sourcing period is quite high. However, when $q_L - q_H$ is large, there is only a small probability that $q_L$ will be selected and hence the size of the prize exerts the main influence on the selection of $I_H$. In such a case, the longer the parallel sourcing period $T$, the lower the optimal investment level chosen by $q_H$. In the case where $(q_H, q_L) = (0.2, 0.45)$, the low quality supplier's optimal investment decreases initially, and then begins to increase for $T > 0.2$, while the high quality supplier's optimal investment decreases for all $T$. This should be expected, for as the parallel sourcing period increases beyond a certain duration, the lower quality supplier is assured the buyer's business for a longer period and is willing to invest more. While the high quality supplier is also assured the buyer's business for a longer duration, his "prize" is decreasing, and hence, so are his optimal investment levels.

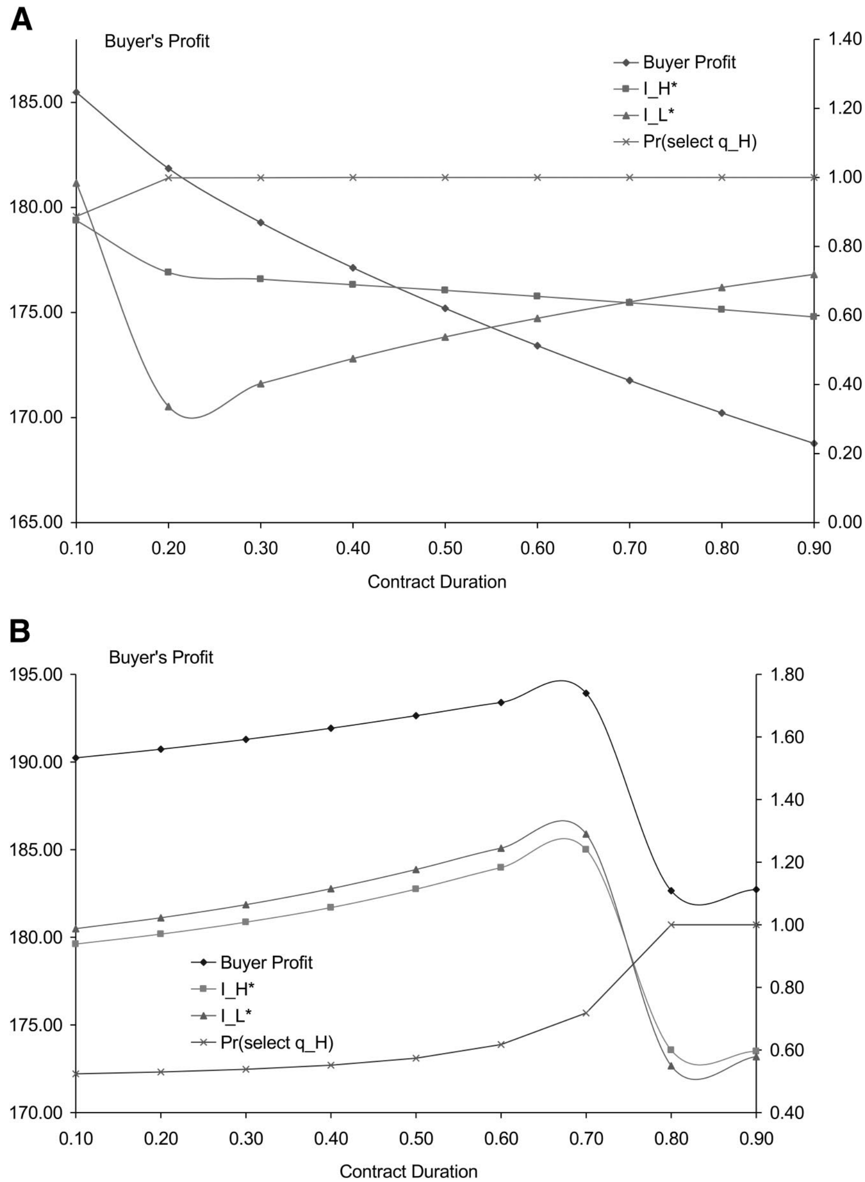
This behavior is contrasted with the case where $(q_H, q_L) = (0.2, 0.23)$. Under such a setting, we observe that the suppliers engage in a "fight for the prize" as $T$ increases. That is, the suppliers continue to undertake more investment as $T$ increases up to 0.7. For all contract durations between 0.1 and 0.7, the probability that the high supplier is selected is slightly above 50% (the solid curve with × in the right panel of Figure 1). We can deduce that the low quality supplier finds it beneficial to continue to invest more at $T$ increases to counteract the reduction in quality "noise" up until $T = 0.7$. This, in turn, encourages fiercer investment from the high quality supplier. For contract durations greater than 0.7, the prize is no longer worth the cost of keeping $\Pr(F_H > F_L)$ low, and the low quality supplier (mimicked by the high quality supplier) significantly decreases his optimal investment level.

One of the most noticeable differences between the two graphs is the optimal duration $T$ (note that $T^*_{q_L=0.45} = 0.1 < 0.7 = T^*_{q_L=0.23}$.) There are several reasons for this result. The buyer is less willing to parallel source for a long duration when one of the suppliers is known to be of a quality much lower than the other. Likewise, with such a large difference between the two quality types, the buyer requires a

[8] We also experimented with other specifications of $C(I)$ and $f(q, I)$ such as $C(I) = A/(U - I)^m - A/U^m$ and $f(I, q) = 1 - q/(1 + I)$ with $A$, $U$, and $m$ being constants. The observations discussed later hold true.

[9] The chosen functional forms of $C(I)$ belong to the family of linear combinations of hyperbolic absolute risk aversion (HARA) functions. HARA functions are commonly used in economic models involving utility functions.

**Figure 1** **Equilibrium Investment, Probability and Buyer Profit for Different Contract Durations.**




much shorter observation period in order to differentiate between the two (with high accuracy). These reasons, combined with the effect of $T$ on the suppliers' investment levels, imply that a buyer will select a much shorter parallel sourcing period when the difference between the two supplier types is "large."

A final observation is that the optimal contract duration does not maximizes the probability that the high supplier is selected in either figure. Waiting until the probability of selecting the high supplier is near 100% is too costly for the buyer, in terms of decreasing the size of the "prize" and increasing the length of time that she sources (with certainty) for a low quality supplier. In addition, note that after $T^*$, both suppliers' optimal investment levels decrease sharply. We observed this behavior in all of our simulation runs.

We are able to summarize our main observations with regards to the optimal investment levels and contract duration in Table 2. Table 2 indicates the effect of increasing $p^S$, $(D/\chi)$, $(q_L - q_H)$, and $\sigma$ on the optimal contract duration and corresponding investment levels, keeping all else equal. [Note: We use the T2

**Table 2** **The Impact on the Buyer's Optimal Contract Duration and Corresponding Equilibrium Suppliers' Investment Levels as Market Parameters Increase**

| | $I_H$ | $I_L$ | $T^*$ |
|---|---|---|---|
| $p^S$ ↑ | ↑ | ↑ | ↑ |
| $(D/\chi)$ ↑ | ↑ | ↑ | ↑ |
| $(q_L - q_H)$ ↑ | ↓ | ↑ or ↓ | ↓ |
| $\sigma$ ↑ | ↓ | ↓ | ↑ |

term decreasing (increasing) to imply non-increasing (non-increasing)].

The first two rows of Table 2 seem fairly intuitive. As either $D$ and/or $p^S$ increase, a supplier's expected profits, both during the parallel sourcing period and after, increase. Therefore, the exercise of equating marginal benefit with marginal cost implies that the suppliers will increase their investment levels for all possible $T$; this, in turn, increases the optimal length of the parallel sourcing period. A similar argument holds when $\chi$ decreases.

The third and fourth rows of Table 2 are worthy of further discussion (Tables 3, 3b, and 4 contain supporting data for these observations). For reasons explained above, as $q_L - q_H$ increases, $T^*$ and $I_H$ decrease. However, the directional impact of increasing $q_L$ (while keeping $q_H$ fixed) on $I_L$ is not as clear. For example, consider the first two rows of Table 4. When $q_L = 0.15$, the probability of winning at $T^* = 0.9$ is high enough to justify "staying in the game" and making a high investment level. As $q_L$ increases from 0.15 to 0.2, the difference in quality remains sufficiently small and the probability of winning sufficiently high for $q_L$ to increase his investment level, while $T^*$ remains the same at 0.9. However, when $q_L = 0.25$, $q_L$ will only "fight for the prize" until $T^* = 0.8$, after which point he significantly drops his investment level. With this shorter parallel sourcing period, his expected profit decreases, and so does his optimal investment level. Hence, an increase in $q_L$ can cause $I_L$ to increase or decrease, depending on the corresponding optimal $T^*$.

Interestingly, while an increase in $\sigma$ causes both $I_H$ and $I_L$ to decrease in equilibrium, it causes $T^*$ to increase. We might expect that a high supplier who is operating in a noisier environment (high $\sigma$) would invest more so as to differentiate himself from the low quality supplier. In fact, we find exactly the opposite to be true. As illustrated in Table 3b, an increase in $\sigma$ (from 0.1 to 1) decreases $q_H$'s marginal benefit of additional investment by increasing the probability that $q_L$ will be selected at $T^* = 0.5$. This leads $q_H$ to decrease his optimal investment level and consequently leads to a decrease in $I_L$ as well. While both suppliers reduce their equilibrium investment levels, they continue to "fight for the prize" for larger values of $T$, implying that it is optimal for the buyer to parallel source for a longer duration.

## 6. Extensions and Discussions

### 6.1. Known Supplier Quality

The previous section illustrated the optimal tournament structure in a setting where the buyer does not know each supplier's quality. A natural question becomes, how does a tournament perform if the buyer knows each supplier's quality and hence can select the high quality at $T = 0$? In such a setting, should a buyer ever commit to using a tournament, where she runs the risk of awarding business to the low quality supplier. Interestingly, we found that the answer is "Yes." Under certain market settings, a buyer is strictly better off running a tournament rather than awarding a sole sourcing contract to the high quality supplier at $T = 0$. This result stems from the competitive pressure a tournament exerts on the high supplier's optimal investment levels.

It is important to point out that in order for the tournament to coax higher investment levels out of the high quality supplier, the buyer must credibly commit to selecting the supplier with the highest *observed* quality. This implies that, with positive probability, the buyer will select the lower quality supplier. While this may seem irrational *ex post*, it is necessary *ex ante* to obtain the desired results.

If the buyer knows who the high quality supplier is, she can offer him a contract of the form $(p^S, 2D, 1)$. Presented with this contract, the high quality supplier will solve Equation (1) where $T = 0$ and $\Pr(F_H \geq F_L) = 1$.

By Assumptions 1 and 3, the high quality supplier's objective function is strictly concave in $I_H$. Thus the supplier's optimal investment level is the unique solution of the following first order condition:

$$2D \cdot p^S \frac{df(q_{H\_only}, I_{H\_only})}{dI_{H\_only}} - C'(I_{H\_only}) = 0. \quad (5)$$

The buyer's profit under optimal investment by the supplier is given by

$$2Df(q_{H\_only}, I^*_{H\_only}) \cdot (p^M - p^S).$$

Observation 1. *When the high quality supplier is offered the contract* $(p^S, 2D, 1)$, *his optimal investment level* $I^*_{H\_only}$ *is always less than the optimal investment level* $I^*_{H\_central}$ *in the case where the investment decision is made centrally by the buyer.*

This observation follows from the standard "double-marginalization" argument. We say that one sourcing strategy dominates another if it yields the buyer a higher expected profit. We summarize below our main results from our numerical experiments.

Observation 2. *A tournament dominates sole sourcing with* $q_H$ *when* $q_H$ *and* $q_L$ *are close, the cost of investment* $\chi$ *is "moderate" and* $p^S$ *is not too low.* (*See Tables 5 to 7 in the Appendix.*)

Observation 3. *If the optimal tournament dominates sole sourcing with* $q_H$ *for some* $\hat{p}^S$, *then it will also dominate for all* $p^S > \hat{p}^S$. (*Tables* 5 *to* 7)

OBSERVATION 4. *If the optimal tournament dominates sole sourcing with $q_H$ for some $\hat{q}_L$, then it will also dominate for all $q_L < \hat{q}_L$. (Table 8)*

OBSERVATION 5. *If the optimal tournament dominates sole sourcing with $q_H$ for some $\hat{\sigma}$, then it will also dominate for all $\sigma < \hat{\sigma}$. (Table 3)*

Our numerical experiments indicate that when presented with the opportunity, a buyer would be strictly better off sourcing from two suppliers of comparably high quality, rather than sole sourcing with the high quality supplier, provided that the cost of quality improving measures is moderate and the revenue per non-defective unit is not too low. Observations (2) to (5) are driven by the competitive pressure parallel sourcing exerts on the high supplier's optimal investment level, thereby assuaging the "double marginalization" effect mentioned above. In all instances when parallel sourcing dominates sole sourcing with $q_H$, $I_H^*$ is significantly greater than $I_{H\_only}$. We can understand this result from the observations made in the previous section, i.e., $I_H^*$ is decreasing in $q_L$. As $q_L$ decreases (i.e., the $q_L$'s quality improves), $q_H$ must invest more so as to improve his chances of being selected at $T$. This increase in $I_H^*$, in turn, makes parallel sourcing more attractive to the buyer. However, it is important to note that the converse is not always true, there are several instances where $I_H^* > I_{H\_only}$ and yet parallel sourcing does not dominate sole sourcing with $q_H$.[10] Fn10

### 6.2. Uncertain Quality

We next consider a setting where the suppliers may be of the same type, i.e., the probability that any one supplier is a high quality supplier is $p_H = \alpha$ and the probability that it is a low quality supplier is $p_L = 1 - \alpha$. Under this scenario, a tournament holds the potential drawback of reducing the optimal investment undertaken by each supplier without the added benefit of differentiating the high quality supplier from the low quality one.

A high quality supplier solves for his optimal investment level by solving a modified form of (1), where the first term is multiplied by $(1 - \alpha)$ and the second multiplied by $\alpha$. Similarly, a low quality supplier solves a modified (2) where the first term is multiplied by $\alpha$ and the second term by $(1 - \alpha)$. Given the suppliers' equilibrium behavior, the buyer selects the optimal contract duration so as to maximize her expected profit:

$$\max_{T\in\hat{T}} \alpha^2(p^M - p^S)2Df(q_H, I_H^*) + 2\alpha(1-\alpha)(p^M - p^S) \times \{[f(q_H, I_H^*) + f(q_L, I_L^*)]DT + [\Pr(F_H^* \geq F_L^*)f(q_H, I_H^*) + (1 - \Pr(F_H^* \geq F_L^*))f(q_L, I_L^*)]2D(1-T)\} + (1-\alpha)^2(p^M - p^S)2Df(q_L, I_L^*). \quad (6)$$

Similar to before, we assume that the suppliers' objective functions have unique global maximum at every $(I_L, T)$ and $(I_H, T)$ in $[0, U) \times \hat{T}$, respectively.[11] Fn11

Given the assumed (general) forms for $\Pr(F_H \geq F_L)$, $f(q, I)$ and $C(I)$, no closed-form expression can be obtained for the suppliers' optimal investment levels from the first order necessary conditions of the suppliers' profit-maximizing problems. We proceed with our equilibrium analysis by numerical simulation, using the same set of parameters as in Section 5. In our simulations, we consider four possible levels/values for $\alpha$: low ($\alpha = 0.25$), low-medium ($\alpha = 0.3$), medium ($\alpha = 0.5$), and high ($\alpha = 0.75$).

We find that observations and insights we obtained in Section 5 in Table 2 hold equally true in the case of unknown qualities. Specifically, the possibility of the two suppliers being the same quality does not change the effect $p^S$, $(D/\chi)$, $(q_L - q_H)$ and $\sigma$ have on the optimal investment levels and contract duration. Representative experiments supporting the observations made in Table 2 can be found in Tables 9 to 13 in the Appendix.

### 6.3. Variable Investment

We next consider the case where there are two different suppliers in the market ($q_H$ and $q_L$) and each supplier selects an investment *rate* at two time points: one at the beginning of the tournament competition $t = 0$ and the other at the end of the tournament and after the sole sourcing contract has been awarded, $t = T$. We denote the investment decision at $t = 0$ and $t = T$ by $I_{i,T}$ and $I_{i,1-T}$ ($i = L, H$), respectively. Given its interpretation as an investment rate, the cost of investment over intervals $[0, T]$ and $[T, 1]$ is given by $T \cdot C(I_{i,T})$ and $(1 - T) \cdot C(I_{i,1-T})$, where $C(I)$ is the investment cost function as before.[12] Fn12

Given a tournament duration of $T$, the high quality supplier solves:

[10] One important remark is that $I_H^*$ is not always greater than $I_{H\_only}$; parallel sourcing fails to motivate the high supplier to exert more effort than if he were the sole supplier when (*i*) the cost of investment $\chi$ is large relative to the potential revenues (determined by $p^S$ and $D$) and the probability of being chosen at $T^*$ is low, and (*ii*) $(q_L - q_H)$ is large, i.e., the low type supplier is of significantly lower quality than the high type supplier.

[11] Again, if the objective functions in (1) and (2) are strictly concave in $I_H$ and $I_L$, then this assumption would hold. Under these assumptions, the equilibrium investment levels $I_H^*$ and $I_L^*$ exist for every $T \in \hat{T}$. This implies the existence of the optimal contract duration $T^*$ since $\hat{T}$ is a finite set. Thus, a pure strategy SPNE of the game, $\{T^*, (I_H^*, I_L^*)\}$, exists. Please see Appendix A for further discussion.

[12] In Section 4, the cost of investment $C(I)$ was a one time event, while it has the interpretation of a cost rate in this section.

$$\max_{I_{H,T}, I_{H,1-T} \in [0,U)} DT(p^S f(q_H, I_{H,T}) - c) - T \cdot C(I_{H,T}) + \Pr(F_H \geq F_L; I_{H,T})\{2D(1-T)[p^S f(q_H, I_{H,1-T}) - c] - (1-T)C(I_{H,1-T})\}. \quad (7)$$

Similarly, the low quality supplier solves:

$$\max_{I_{L,T}, I_{L,1-T} \in [0,U)} DT(p^S f(q_L, I_{L,T}) - c) - T \cdot C(I_{L,T}) + \Pr(F_L \geq F_H; I_{L,T})\{2D(1-T)[p^S f(q_L, I_{L,1-T}) - c] - (1-T)C(I_{L,1-T})\}. \quad (8)$$

Given the suppliers' equilibrium behavior, the buyer selects the tournament duration $T$ that maximizes her expected profit:

$$\max_{T \in \hat{T}} (p^M - p^S)\{DT[f(q_H, I^*_{H,T}) + f(q_L, I^*_{L,T})] + 2D(1-T) \times [\Pr(F^*_H \geq F^*_L; I^*_{H,T}, I^*_{L,T}) \cdot f(q_H, I^*_{H,1-T}) + (1 - \Pr(F^*_H \geq F^*_L; I^*_{H,T}, I^*_{L,T})) \cdot f(q_L, I^*_{L,1-T})]\}. \quad (9)$$

Observation 6. *The optimal investment rate $I_{i,1-T}$ $(i = L, H)$ is independent of $T$, $(q_L - q_H)$ and $\sigma$.*

Note that both the benefits and costs of investment rate $I_{i,1-T}$ are proportionally scaled by $(1 - T)$ and hence $T$ does not affect the suppliers' optimal investment rate. Similarly, a competitor's quality level and noise in the system have no effect on a supplier's optimal investment level once he has been selected as the sole source supplier. The same cannot be said for $I_{i,T}$, i.e., $I_{i,T}$ is a function of $T$, $(q_L - q_H)$ and $\sigma$ since all three market parameters directly impact the probability that the high quality supplier will be selected.

Observation 7. *$I_{i,1-T}$ $(i = L, H)$ is an increasing function of $p^S$ and $(D/\chi)$.*

This observation is by inspection of (7) and (8). We cannot, however, make a similar extrapolation for $I_{i,T}$; as in Section 5, we must turn to numerical simulations in order to gain further insights. Using the same parameter setting as in Section 5, we examine the optimal contract duration for the buyer and how the suppliers' optimal investment levels $I_{i,T}$ are affected by $T$,
Fn13 and other market parameters.[13]

We found that the relationships in Table 2 continue to hold true in the case of variable investment, with the notable exception of the relationship be-
T2b tween $(q_L - q_H)$ and $I_{H,T}$ (please see Table 2b below). While in the case of irreversible and fixed investment, an increase in quality difference induced the high type supplier to invest less, the same is not true in the case of variable investment. We found that a high quality supplier may actually increase his investment rate over the tournament period, due to the "reversible" nature of investment. As with a fixed investment, an increase in $I_{H,T}$ increases the high quality supplier's probability of winning the tournament (the marginal benefit); however, the marginal cost of such an increase is less than in the fixed investment case (since it is reversible at $T$). As a result, we found that for a wide range of market settings, $I_{H,T}$ is increasing in $(q_L - q_H)$.

In addition, we found that,

**Table 2b** **Relationship Between Market Parameters, Equilibrium Investment and *T** Under Variable Investment**

| | $I_H$ | $I_L$ | $T^*$ |
|---|---|---|---|
| $p^S$ ↑ | ↑ | ↑ | ↑ |
| $(D/\chi)$ ↑ | ↑ | ↑ | ↑ |
| $(q_L - q_H)$ ↑ | ↓ or ↑ | ↑ or ↓ | ↓ |
| $\sigma$ ↑ | ↓ | ↓ | ↑ |

Observation 8. *Similar to the case with one-time irreversible investment, if a tournament dominates sole sourcing with supplier $q_H$ in a market setting with $\hat{q}_L$, $\hat{\sigma}$, and $\hat{p}^S$, then it will also dominate for all $q_L < \hat{q}_L$, $\sigma < \hat{\sigma}$, and $p^S > \hat{p}^S$.*

Given the observation above, one might then wonder whether the market settings under which a tournament dominates sole sourcing is the same under reversible and irreversible investment. We found that the answer is no; a tournament dominates sole sourcing under a wider range of settings when investment is irreversible. The reason for this is intuitive: the ability of a supplier to change his investment level at time $T$ weakens the competitive benefits of a tournament. The reversible nature of investment implies that higher levels of investment coaxed from suppliers during the tournament are not carried over to the sole sourcing period, and hence, the buyer's expected profits are reduced. Following this line of reasoning, we also found that the optimal tournament duration $T$ is never shorter under variable investment than irreversible investment case, i.e., the buyer has the incentive to dual source for a longer period of time in order to maintain high investment levels longer.

## 7. Managerial Insights and Future Research

When a buyer faces a set of suppliers of unknown quality and nonverifiable investments, she must design a sourcing arrangement that allows her to both extract some information about the suppliers and

[13] In the interest of space limitations, we have omitted the supporting tables from this section. Interested readers can contact the authors.

encourage (costly) investments on their part. The benefits of a tournament are two-pronged: (1) it gives the buyer a chance to observe and learn more about the suppliers' qualities; and (2) it motivates suppliers to undertake costly investment by promising the supplier who delivers the higher quality level the entirety of the buyer's business once the parallel sourcing period is over, in effect offering the winning supplier a prize. In this paper, we seek to establish the optimal tournament duration under various market settings. Via our numerical experiments, we were able to characterize the optimal duration of the parallel sourcing period as a function of the supplier characteristics. In addition, we found that parallel sourcing dominates sole sourcing with the high quality supplier under a variety of supplier settings.

Our simulations demonstrate that when the cost of investment is relatively small, a buyer can coax high investment levels out of the high quality supplier without competition; that in combination with the loss in quality (and hence profit) arising from (possibly) sourcing from a low quality supplier make a tournament unattractive to the buyer. On the other hand, when the cost of investment is very high, the presence of competition may make the high quality supplier invest even less than if it were the sole supplier. The greatest benefit from a tournament occurs when the supplier's return on investment ($p^S/\chi$) are moderate and the loss in quality associated with sourcing from $q_L$ is not very large. In addition, we found that the effectiveness of a tournament increases when the suppliers' investments are irreversible. We also observed that when the suppliers are of comparably high quality, it is optimal to parallel source for almost the entire contract duration. Furthermore, the benefits of parallel sourcing over sole sourcing increase as the noise in the supply chain, i.e., $\sigma$, decreases. This is driven by the fact that investment levels over the tournament duration are decreasing in $\sigma$. A smaller $\sigma$ induces the suppliers to increase their investment levels and fight for the prize, yielding a higher realized quality level, and hence profit, for the buyer.

Evidence for our theoretical and numerical results can be found in industry practices in electronics manufacturing and aerospace. Both Solectron (a major electronics manufacturing firm) and the Department of Defense have successfully employed tournaments. As noted by participants in the DOD's parallel sourcing arrangement, "the most important lesson to draw from the engine experience is the value of competition. Competition is the only sure way to get the best effort." (Drewes 1987, p. 151).

From our discussions with various companies, we found that parallel sourcing via a tournament is an especially attractive sourcing strategy when a buyer wishes to "prod" a lethargic incumbent supplier into taking quality improving actions. For example, Solectron, used a tournament to procure a cable commodity with two suppliers, one of which was an incumbent firm (Caltabiano 2001). The decision to run a tournament was made after Solectron doubted that its incumbent was providing it with a high quality product at a reasonable price. Solectron was able to effectively parallel source due to the presence of an alternative supplier of "comparable" quality. Similarly, what allowed the Department of Defense's experience with General Electric and Pratt and Whitney to be a great success was the lack of large disparity between the two companies' production abilities. In the case of Solectron, the credible threat from the competing non-incumbent provided a strong incentive for the incumbent to invest more in maintaining or improving the high level of product quality. As a result, the incumbent won the tournament and hence the remainder of Solectron's business. In fact, most companies with whom we spoke echoed a similar story. Incumbent suppliers have an advantage relative to new suppliers with regards to their familiarity with the buyer's demands. This knowledge often allows them to provide the buyer with a higher level of service. While in most cases the incumbent suppliers eventually win the tournament, the threat of losing business to an identified alternative supplier raises the performance of the incumbent supplier and quality of his product.

The results of this research naturally lead to many interesting questions. In this paper, we assumed that the buyer knows the possible quality levels of her potential suppliers. However, in some situations, the buyer has relatively little information about the suppliers' qualities. In order to remedy her lack of information, a buyer often will undertake costly actions so as to improve her information about a supplier's quality. For example, it is commonplace for Solectron to send a team out to conduct site visits at a potential supplier's site. These site visits are costly to Solectron; therefore, it would be of great value to be able to quantify the value of such information. That is, if a buyer decides to parallel source, what is the value of knowing what types of suppliers she faces? This paper took a step in the direction of answering this question by considering the case where the buyer is not certain if the suppliers are of the same or different quality levels. It would be interesting to carry this relaxation further and consider the case where the buyer only knows that a supplier's quality has some probabilistic distribution.

Another interesting extension is related to the fact that the suppliers in our model are asymmetric only in their intrinsic quality attribute $q$ and are assumed to be symmetric (in the sense of sharing common forms) in their "technology" function $f(q, I)$ and the cost function $C(I)$. Here we interpret $f(q, I)$ as a technology function transforming $q$ and $I$ into some productivity measure such as the quality of the final outputs. The suppliers may well be different in the way that they convert intrinsic quality and investments into true productivity measures and in their investment cost structures. Namely, the functions $f(q, I)$ and $C(I)$ may be of different forms for different suppliers and the functional forms are private information to the suppliers. While we would expect that the increasing amount of private information possessed by potential asymmetric suppliers would make a tournament a more favorable choice than any other traditional sourcing schemes to a buyer, it is another area for future research.

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**Appendix**

## A. Proof of Existence of Pure Strategy Equilibrium in Section 4

We first introduce some notations and definitions. Let $h_1(I_H, I_L, T)$ and $h_2(I_L, I_H, T)$ denote the objective functions in (1) and (2), respectively.

$$h_1(I_H, I_L, T) \equiv DT(p^S f(q_H, I_H) - c) + 2D(1 - T)\Pr(F_H \geq F_L)(p^S f(q_H, I_H) - c) - C(I_H)$$

$$h_2(I_L, I_H, T) \equiv DT(p^S f(q_L, I_L) - c) + 2D(1 - T)\Pr(F_L > F_H)(p^S f(q_L, I_L) - c) - C(I_L)$$

Define correspondences $g_1(I_L, T)$ and $g_2(I_H, T)$ as

$$g_1(I_L, T) = \arg\max_{I_H \in [0,U)} h_1(I_H, I_L, T)$$

$$g_2(I_H, T) = \arg\max_{I_L \in [0,U)} h_2(I_L, I_H, T) \quad (10)$$

For every $(I_L, T) \in [0, U] \times \hat{T}$, $h_1(I_H, I_L, T)$ is a continuous function of $I_H$ over interval $[0, U)$ and

$$\lim_{I_H \to U} h_1(I_H, I_L, T) = -\infty \quad (11)$$

by Assumption 1. Thus $g_1(I_L, T)$ is well-defined and nonempty and so is $g_2(I_H, T)$.

LEMMA 5. *If a correspondence $\Gamma : X \to Y$ is upper hemi-continuous and single-valued, then it is continuous.*

A direct application of the Theorem of Maximum (p. 62, Stokey and Lucas 1989) and Lemma 5 to $g_1$ and $g_2$ yields the following lemma.

LEMMA 6. *Under Assumptions* 1 *to* 4, *the correspondences $g_1$ and $g_2$ defined by* (10) *are single-valued continuous functions. This claim is also true when replacing $h_1$ and $h_2$ in* (10) *with the respective suppliers' objective functions in the extension cases discussed in Section* 6.

With the help of Lemmas 5 and 6, we establish the existence of pure strategy Nash equilibrium $(I_H^*, I_L^*)$ for any given contract $(p^S, D, T)$ through Proposition 1 by applying the Brouwer Fixed-Point Theorem (see Chapter 6 of Border 1985).

*Proof of Proposition 1.* We begin with the setting of known quality levels. Let $V \equiv [0, U] \subset R$ and $Y = V \times V = V^2$. For any given $T \in \hat{T}$, we define a mapping $\Gamma \equiv (\Gamma_1, \Gamma_2) : Y \to R^2$ as follows.

$$\begin{cases} \Gamma_1(I_H, I_L) = g_1(I_L, T) \\ \Gamma_2(I_H, I_L) = g_2(I_H, T) \end{cases}$$

where $g_i$ $(i = 1, 2)$ are given by (10). The equilibrium investment levels $(I_H^*, I_L^*)$ that simultaneously maximize suppliers' respective objectives are thus given by a fixed point of mapping $\Gamma$ in its domain $Y$. In what follows, we show that $\Gamma$ indeed has a fixed point in $Y$. $\Gamma$ is continuous by Lemma 6. For any $(I_H, I_L) \in Y$, we have $g_1(I_L, T) \in [0, U)$ and $g_2(I_H, T) \in [0, U)$ by definition. So, $\Gamma : Y \to Y$. Applying the Brouwer Fixed-Point Theorem, we conclude that $\Gamma$ has a fixed point in $Y$. The above arguments can be replicated for the extension cases such as the one with unknown valuations (with the objective functions as defined in Section 6.2). □

Since the set $\hat{T}$ is a finite set, the objective function of the buyer (3), denoted by $\pi_{\text{buyer}}(T)$, takes on a finite number of values in $\hat{T}$. Both the maximum and the maximizer $T^*$ of $\pi_{\text{buyer}}(T)$ exist. Therefore, this proves the existence of a pure strategy SPNE $\{T^*, (I_H^*, I_L^*)\}$ to our Stackelberg game for the case of known valuations; with the argument for the extension cases in Section 6 (e.g., the case of unknown supplier types) being the same.

Assumptions 1 to 4 guarantee the existence of a pure strategy SPNE. However, they do not imply anything regarding the uniqueness of the SPNE strategy. Note that the uniqueness of the SPNE depends on the uniqueness of both $(I_H^*, I_L^*)$ and $T^*$. A sufficient condition for $(I_H^*, I_L^*)$ to be unique is that the mapping $(\Gamma_1, \Gamma_2)$ defined in the proof of Proposition 1 is a contraction mapping. On the other hand, a sufficient condition for the uniqueness of $T^*$ is that $\pi_{\text{buyer}}(T)$ is strictly unimodal over interval [0, 1]. Given these two conditions, the SPNE strategy $\{T^*, (I_H^*, I_L^*)\}$ is unique. Unfortunately, it is usually not an easy task to verify that $(\Gamma_1, \Gamma_2)$ is a contraction mapping. For instance, while Assumptions 1 to 4 are satisfied in all of our numerical examples presented in the next section, it is very difficult to check the contraction mapping condition is satisfied in those examples. Nevertheless, the SPNE strategy in each of those examples seems to be quite robust in the sense that it converges to the same solution regardless which initial values that we use for computing the equilibrium strategies $(I_H^*, I_L^*)$ and $T^*$.

T13

# B. Tables

**Table 3 Simulation Output with $q_H = 0.1$, $q_L = 0.15$, $p^S = 4$ and $D = 100$**

| $\sigma$ | $\chi$ | $T^*$ | $I_H^*$ | $I_L^*$ | $\Pr(F_H \geq F_L)$ | $\pi_{Buyer}$ | $I_{H\_only}$ | $\pi_{Buyer\_H\_only}$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 1 | 0.9 | 1.529 | 1.72 | 0.541 | 199.1 | 1.522 | 199.4 |
| 1 | 1 | 0.9 | 1.764 | 1.968 | 0.665 | 199.5 | 1.522 | 199.4 |
| 0.5 | 1 | 0.9 | 1.85 | 2.044 | 0.811 | 199.6 | 1.522 | 199.4 |
| 0.1 | 1 | 0.8 | 1.904 | 2.085 | 0.579 | 199.6 | 1.522 | 199.4 |
| 5 | 10 | 0.9 | 1.03 | 1.202 | 0.606 | 197.5 | 1.008 | 198 |
| 1 | 10 | 0.9 | 1.165 | 1.353 | 0.601 | 198.2 | 1.008 | 198 |
| 0.5 | 10 | 0.9 | 1.296 | 1.501 | 0.672 | 198.6 | 1.008 | 198 |
| 0.1 | 10 | 0.8 | 1.473 | 1.687 | 0.656 | 199.1 | 1.008 | 198 |
| 5 | 50 | 0.9 | 0.674 | 0.794 | 0.745 | 194.6 | 0.65 | 195.5 |
| 1 | 50 | 0.9 | 0.803 | 0.949 | 0.728 | 196 | 0.65 | 195.5 |
| 0.5 | 50 | 0.8 | 0.804 | 0.951 | 0.599 | 196 | 0.65 | 195.5 |
| 0.1 | 50 | 0.7 | 1.039 | 1.222 | 0.647 | 197.7 | 0.65 | 195.5 |
| 5 | 100 | 0.9 | 0.507 | 0.591 | 0.842 | 192.1 | 0.507 | 193.8 |
| 1 | 100 | 0.8 | 0.551 | 0.655 | 0.582 | 192.9 | 0.507 | 193.8 |
| 0.5 | 100 | 0.8 | 0.659 | 0.786 | 0.642 | 194.5 | 0.507 | 193.8 |
| 0.1 | 100 | 0.7 | 0.931 | 0.981 | 0.804 | 196.7 | 0.507 | 193.8 |

**Table 3b Suppliers' Equilibrium Investment for $q_H = 0.1$, $q_L = 0.3$, $\chi = 10$, $p^S = 3$, and $D = 100$ when vary $\sigma$**

| | $\sigma = 0.1$ | | | $\sigma = 1$ | | |
|---|---|---|---|---|---|---|
| $T$ | $I_H$ | $I_L$ | $\Pr(F_H > F_L)$ | $I_H$ | $I_L$ | $\Pr(F_H \geq F_L)$ |
| 0.1 | 1.077 | 1.549 | 0.586 | 0.819 | 1.218 | 0.507 |
| 0.2 | 1.095 | 1.578 | 0.608 | 0.829 | 1.229 | 0.510 |
| 0.3 | 1.116 | 1.611 | 0.638 | 0.844 | 1.246 | 0.516 |
| 0.4 | 1.137 | 1.647 | 0.686 | 0.859 | 1.265 | 0.524 |
| 0.5 | 1.151 | 1.676 | 0.766 | 0.873 | 1.284 | 0.535 |
| 0.6 | 0.995 | 1.045 | 0.999 | 0.891 | 1.309 | 0.554 |
| 0.7 | 0.845 | 1.075 | 1.000 | 0.917 | 1.349 | 0.597 |
| 0.8 | 0.827 | 1.116 | 1.000 | 0.953 | 1.414 | 0.720 |
| 0.9 | 0.821 | 1.151 | 1.000 | 0.808 | 1.152 | 1.000 |

**Table 4 Simulation Output for Suppliers when $p^S = 3$, $\sigma = 1$, $\chi = 10$ and $D = 100$**

| $q_H$ | $q_L$ | $T^*$ | $I_H$ | $I_L$ | $\Pr(F_H \geq F_L)$ | $\pi_{Buyer}$ |
|---|---|---|---|---|---|---|
| 0.1 | 0.15 | 0.9 | 1.084 | 1.265 | 0.620 | 395.7 |
| 0.1 | 0.2 | 0.9 | 1.041 | 1.352 | 0.805 | 393.8 |
| 0.1 | 0.25 | 0.8 | 0.961 | 1.341 | 0.638 | 390.3 |
| 0.1 | 0.3 | 0.8 | 0.953 | 1.414 | 0.720 | 387.6 |
| 0.1 | 0.35 | 0.8 | 0.933 | 1.444 | 0.825 | 384.0 |
| 0.1 | 0.4 | 0.8 | 0.887 | 1.388 | 0.939 | 378.5 |
| 0.1 | 0.45 | 0.7 | 0.911 | 1.480 | 0.774 | 374.1 |
| 0.1 | 0.5 | 0.7 | 0.900 | 1.467 | 0.857 | 368.5 |

**Table 5 Simulation Output for Suppliers when $p^S = 3$, $\sigma = 0.1$, and $D = 100$**

| $q^H$ | $q^L$ | $\chi$ | $T^*$ | $I_H$ | $I_L$ | $\Pr(F_H \geq F_L)$ | $\pi_{\text{Buyer}}$ | $I_{H\_only}$ | $\pi_{\text{Buyer\_H\_only}}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.2 | 1 | 0.8 | 1.805 | 2.131 | 0.755 | 398.5 | 1.461 | 398.6 |
| 0.1 | 0.2 | 10 | 0.7 | 1.274 | 1.627 | 0.723 | 396.3 | 0.943 | 395.4 |
| 0.1 | 0.2 | 50 | 0.6 | 0.855 | 1.132 | 0.761 | 391.5 | 0.589 | 389.7 |
| 0.1 | 0.2 | 100 | 0.5 | 0.688 | 0.892 | 0.738 | 387.4 | 0.451 | 385.8 |
| 0.2 | 0.3 | 1 | 0.7 | 2.026 | 2.192 | 0.834 | 394.7 | 1.779 | 395.4 |
| 0.2 | 0.3 | 10 | 0.6 | 1.523 | 1.731 | 0.791 | 390 | 1.21 | 388.6 |
| 0.2 | 0.3 | 50 | 0.4 | 1.039 | 1.198 | 0.718 | 380.1 | 0.777 | 377.1 |
| 0.2 | 0.3 | 100 | 0.4 | 0.833 | 0.932 | 0.802 | 373.3 | 0.601 | 369.6 |
| 0.3 | 0.4 | 1 | 0.5 | 2.138 | 2.229 | 0.765 | 386.6 | 1.96 | 388.7 |
| 0.3 | 0.4 | 10 | 0.4 | 1.652 | 1.773 | 0.749 | 378.3 | 1.39 | 377.5 |
| 0.3 | 0.4 | 50 | 0.3 | 1.153 | 1.203 | 0.788 | 363.1 | 0.905 | 359.6 |
| 0.3 | 0.4 | 100 | 0.2 | 0.914 | 0.914 | 0.781 | 352 | 0.701 | 348.4 |

**Table 6 Simulation Output for Suppliers when $p^S = 4$, $\sigma = 0.1$, and $D = 100$**

| $q^H$ | $q^L$ | $\chi$ | $T^*$ | $I_H$ | $I_L$ | $\Pr(F_H \geq F_L)$ | $\pi_{\text{Buyer}}$ | $I_{H\_only}$ | $\pi_{\text{Buyer\_H\_only}}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.2 | 1 | 0.8 | 1.879 | 2.185 | 0.736 | 199.3 | 1.522 | 199.4 |
| 0.1 | 0.2 | 10 | 0.7 | 1.363 | 1.721 | 0.692 | 198.4 | 1.008 | 198 |
| 0.1 | 0.2 | 50 | 0.6 | 0.949 | 1.253 | 0.713 | 196.5 | 0.65 | 195.5 |
| 0.1 | 0.2 | 100 | 0.5 | 0.756 | 1.01 | 0.693 | 194.7 | 0.507 | 193.8 |
| 0.2 | 0.3 | 1 | 0.7 | 2.092 | 2.244 | 0.819 | 197.6 | 1.84 | 197.9 |
| 0.2 | 0.3 | 10 | 0.6 | 1.624 | 1.838 | 0.76 | 195.6 | 1.287 | 195 |
| 0.2 | 0.3 | 50 | 0.4 | 1.144 | 1.328 | 0.685 | 191.4 | 0.853 | 189.9 |
| 0.2 | 0.3 | 100 | 0.4 | 0.952 | 1.097 | 0.745 | 188.8 | 0.673 | 186.5 |
| 0.3 | 0.4 | 1 | 0.6 | 2.176 | 2.251 | 0.891 | 193.6 | 2.015 | 194.7 |
| 0.3 | 0.4 | 10 | 0.5 | 1.752 | 1.863 | 0.836 | 190.3 | 1.472 | 189.8 |
| 0.3 | 0.4 | 50 | 0.3 | 1.28 | 1.373 | 0.745 | 183.9 | 0.992 | 181.8 |
| 0.3 | 0.4 | 100 | 0.3 | 1.046 | 1.075 | 0.816 | 179.4 | 0.785 | 176.7 |

**Table 7 Simulation Output for Suppliers when $p^S = 2$, $\sigma = 0.1$, and $D = 100$**

| $q^H$ | $q^L$ | $\chi$ | $T^*$ | $I_H$ | $I_L$ | $\Pr(F_H \geq F_L)$ | $\pi_{\text{Buyer}}$ | $I_{H\_only}$ | $\pi_{\text{Buyer\_H\_only}}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.1 | 0.2 | 1 | 0.8 | 1.669 | 2.047 | 0.798 | 597.2 | 1.372 | 597.5 |
| 0.1 | 0.2 | 10 | 0.7 | 1.109 | 1.434 | 0.801 | 592.4 | 0.851 | 591.5 |
| 0.1 | 0.2 | 50 | 0.5 | 0.68 | 0.896 | 0.74 | 581.4 | 0.507 | 581.3 |
| 0.1 | 0.2 | 100 | 0.4 | 0.51 | 0.662 | 0.74 | 573.3 | 0.377 | 574.8 |
| 0.2 | 0.3 | 1 | 0.7 | 1.897 | 2.074 | 0.867 | 590.8 | 1.689 | 592.1 |
| 0.2 | 0.3 | 10 | 0.5 | 1.333 | 1.523 | 0.727 | 580.3 | 1.1 | 579.6 |
| 0.2 | 0.3 | 50 | 0.4 | 0.846 | 0.921 | 0.814 | 560.3 | 0.673 | 559.4 |
| 0.2 | 0.3 | 100 | 0.3 | 0.639 | 0.651 | 0.817 | 546.1 | 0.505 | 546.8 |
| 0.3 | 0.4 | 1 | 0.5 | 2.035 | 2.131 | 0.785 | 577.8 | 1.877 | 581.2 |
| 0.3 | 0.4 | 10 | 0.4 | 1.468 | 1.541 | 0.806 | 560.5 | 1.27 | 561 |
| 0.3 | 0.4 | 50 | 0.2 | 0.929 | 0.873 | 0.807 | 528.7 | 0.785 | 530 |
| 0.3 | 0.4 | 100 | 0.1 | 0.696 | 0.549 | 0.828 | 508.4 | 0.589 | 511.4 |

**Table 8 Simulation Runs with $\sigma = 0.1$, $\chi = 75$, $p^S = 4$ and $D = 100$**

| $q_H$ | $q_L$ | $T^*$ | $I_H^*$ | $f(q_H, I_H^*)$ | $I_L^*$ | $f(q_L, I_L^*)$ | $\Pr(H\_win)$ | $\pi_{\text{Buyer}}$ | $I_{H\_only}$ | $\pi_{\text{Buyer\_H\_only}}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.2 | 0.25 | 0.6 | 1.143 | 0.968 | 1.193 | 0.952 | 0.756 | 192.4 | 0.747 | 188 |
| 0.2 | 0.3 | 0.4 | 1.034 | 0.962 | 1.198 | 0.929 | 0.717 | 190 | 0.747 | 188 |
| 0.2 | 0.35 | 0.3 | 0.972 | 0.958 | 1.166 | 0.897 | 0.78 | 187.9 | 0.747 | 188 |
| 0.2 | 0.4 | 0.2 | 0.921 | 0.955 | 1.104 | 0.855 | 0.834 | 186.3 | 0.747 | 188 |
| 0.2 | 0.45 | 0.1 | 0.875 | 0.951 | 0.984 | 0.795 | 0.887 | 185.5 | 0.747 | 188 |

**Table 9** Simulation Output with $\alpha = 0.25$, $q_H = 0.1$, $q_L = 0.15$, $p^S = 4$, and $D = 100$

| $\sigma$ | $\chi$ | $T^*$ | $I_H^*$ | $I_L^*$ | $\Pr(F_H \geq F_L)$ | $\pi_{\text{Buyer}}$ | $\tilde{I}_{\text{Average}}$ | $\tilde{\pi}_{\text{Buyer\_certain}}$ |
|---|---|---|---|---|---|---|---|---|
| 3 | 1 | 0.95 | 1.557 | 1.647 | 0.601 | 198.87 | 1.626 | 199.07 |
| 1 | 1 | 0.95 | 1.686 | 1.748 | 0.800 | 199.08 | 1.626 | 199.07 |
| 0.5 | 1 | 0.9 | 1.709 | 1.761 | 0.637 | 199.11 | 1.626 | 199.07 |
| 0.1 | 1 | 0.8 | 1.854 | 1.888 | 0.640 | 199.31 | 1.626 | 199.07 |
| 3 | 10 | 0.95 | 1.034 | 1.085 | 0.808 | 196.68 | 1.096 | 197.29 |
| 1 | 10 | 0.9 | 1.105 | 1.138 | 0.713 | 197.03 | 1.096 | 197.29 |
| 0.5 | 10 | 0.9 | 1.128 | 1.142 | 0.910 | 197.08 | 1.096 | 197.29 |
| 0.1 | 10 | 0.7 | 1.314 | 1.315 | 0.678 | 197.93 | 1.096 | 197.29 |
| 3 | 50 | 0.9 | 0.620 | 0.654 | 0.646 | 192.31 | 0.713 | 194.07 |
| 1 | 50 | 0.8 | 0.657 | 0.674 | 0.604 | 192.66 | 0.713 | 194.07 |
| 0.5 | 50 | 0.8 | 0.744 | 0.734 | 0.722 | 193.57 | 0.713 | 194.07 |
| 0.1 | 50 | 0.6 | 0.892 | 0.855 | 0.737 | 195.03 | 0.713 | 194.07 |
| 3 | 100 | 0.9 | 0.481 | 0.497 | 0.702 | 189.62 | 0.559 | 191.86 |
| 1 | 100 | 0.8 | 0.517 | 0.518 | 0.644 | 190.10 | 0.559 | 191.86 |
| 0.5 | 100 | 0.8 | 0.583 | 0.555 | 0.808 | 190.96 | 0.559 | 191.86 |
| 0.1 | 100 | 0.5 | 0.704 | 0.653 | 0.714 | 192.68 | 0.559 | 191.86 |

**Table 10** Simulation Output for Suppliers when $\alpha = 0.5$, $p^S = 2$, $\sigma = 0.1$, and $D = 100$

| $q^H$ | $q^L$ | $\chi$ | $T^*$ | $I_H$ | $I_L$ | $\Pr(F_H \geq F_L)$ | $\pi_{\text{Buyer}}$ |
|---|---|---|---|---|---|---|---|
| 0.1 | 0.2 | 1 | 0.8 | 1.528 | 1.879 | 0.856 | 596.3 |
| 0.1 | 0.2 | 10 | 0.6 | 0.960 | 1.252 | 0.717 | 589.1 |
| 0.1 | 0.2 | 50 | 0.5 | 0.562 | 0.728 | 0.815 | 574.8 |
| 0.1 | 0.2 | 100 | 0.4 | 0.412 | 0.523 | 0.802 | 565.1 |
| 0.2 | 0.3 | 1 | 0.6 | 1.794 | 1.989 | 0.724 | 588.9 |
| 0.2 | 0.3 | 10 | 0.5 | 1.193 | 1.360 | 0.774 | 574.9 |
| 0.2 | 0.3 | 50 | 0.3 | 0.714 | 0.784 | 0.763 | 548.9 |
| 0.2 | 0.3 | 100 | 0.2 | 0.527 | 0.549 | 0.762 | 532.2 |
| 0.3 | 0.4 | 1 | 0.5 | 1.940 | 2.043 | 0.802 | 574.3 |
| 0.3 | 0.4 | 10 | 0.3 | 1.322 | 1.402 | 0.744 | 551.0 |
| 0.3 | 0.4 | 50 | 0.2 | 0.798 | 0.756 | 0.821 | 512.1 |
| 0.3 | 0.4 | 100 | 0.1 | 0.587 | 0.509 | 0.810 | 488.9 |

**Table 11** Simulation Output for Suppliers when $\alpha = 0.5$, $p^S = 3$, $\sigma = 0.1$, and $D = 100$

| $q^H$ | $q^L$ | $\chi$ | $T^*$ | $I_H$ | $I_L$ | $\Pr(F_H \geq F_L)$ | $\pi_{\text{Buyer}}$ |
|---|---|---|---|---|---|---|---|
| 0.1 | 0.2 | 1 | 0.8 | 1.681 | 2.027 | 0.794 | 398.1 |
| 0.1 | 0.2 | 10 | 0.7 | 1.130 | 1.461 | 0.788 | 394.9 |
| 0.1 | 0.2 | 50 | 0.5 | 0.704 | 0.937 | 0.721 | 387.7 |
| 0.1 | 0.2 | 100 | 0.5 | 0.540 | 0.708 | 0.822 | 382.6 |
| 0.2 | 0.3 | 1 | 0.7 | 1.916 | 2.096 | 0.861 | 393.7 |
| 0.2 | 0.3 | 10 | 0.5 | 1.360 | 1.562 | 0.715 | 386.9 |
| 0.2 | 0.3 | 50 | 0.4 | 0.891 | 1.015 | 0.772 | 374.1 |
| 0.2 | 0.3 | 100 | 0.3 | 0.686 | 0.766 | 0.762 | 364.8 |
| 0.3 | 0.4 | 1 | 0.5 | 2.053 | 2.155 | 0.778 | 384.7 |
| 0.3 | 0.4 | 10 | 0.4 | 1.509 | 1.619 | 0.780 | 373.5 |
| 0.3 | 0.4 | 50 | 0.2 | 0.988 | 1.033 | 0.746 | 353.2 |
| 0.3 | 0.4 | 100 | 0.2 | 0.763 | 0.746 | 0.813 | 339.8 |

**Table 12** **Simulation Output for Suppliers when $\alpha = 0.5$, $p^S = 4$, $\sigma = 0.1$, and $D = 100$**

| $q^H$ | $q^L$ | $\chi$ | $T^*$ | $I_H$ | $I_L$ | $\Pr(F_H \geq F_L)$ | $\pi_{\text{Buyer}}$ |
|---|---|---|---|---|---|---|---|
| 0.1 | 0.2 | 1 | 0.8 | 1.762 | 2.095 | 0.768 | 199.2 |
| 0.1 | 0.2 | 10 | 0.7 | 1.226 | 1.573 | 0.743 | 197.9 |
| 0.1 | 0.2 | 50 | 0.6 | 0.808 | 1.071 | 0.788 | 195.1 |
| 0.1 | 0.2 | 100 | 0.5 | 0.631 | 0.843 | 0.758 | 192.9 |
| 0.2 | 0.3 | 1 | 0.7 | 1.992 | 2.162 | 0.842 | 197.1 |
| 0.2 | 0.3 | 10 | 0.6 | 1.474 | 1.678 | 0.806 | 194.4 |
| 0.2 | 0.3 | 50 | 0.4 | 0.997 | 1.154 | 0.729 | 189.0 |
| 0.2 | 0.3 | 100 | 0.4 | 0.794 | 0.894 | 0.810 | 185.1 |
| 0.3 | 0.4 | 1 | 0.5 | 2.114 | 2.210 | 0.768 | 192.8 |
| 0.3 | 0.4 | 10 | 0.4 | 1.614 | 1.738 | 0.754 | 188.1 |
| 0.3 | 0.4 | 50 | 0.3 | 1.116 | 1.180 | 0.786 | 179.8 |
| 0.3 | 0.4 | 100 | 0.2 | 0.884 | 0.911 | 0.771 | 173.8 |

**Table 13** **Simulation Output for $\alpha = 0.3$, $q_H = 0.2$, $q_L = 0.3$, $\sigma = 0.1$ and $p^S = 3$**

| $\chi$ | $D$ | $T^*$ | $I_H$ | $I_L$ | $\Pr(F_H \geq F_L)$ | $\pi_{\text{Buyer}}$ |
|---|---|---|---|---|---|---|
| 1 | 10 | 0.5 | 1.399 | 1.441 | 0.786 | 38.3 |
| 10 | 10 | 0.2 | 0.715 | 0.659 | 0.762 | 35.7 |
| 50 | 10 | 0.1 | 0.328 | 0.242 | 0.811 | 32.8 |
| 100 | 10 | 0.1 | 0.210 | 0.139 | 0.828 | 31.7 |
| 1 | 100 | 0.6 | 1.958 | 2.035 | 0.732 | 392.0 |
| 10 | 100 | 0.5 | 1.399 | 1.441 | 0.786 | 383.4 |
| 50 | 100 | 0.3 | 0.917 | 0.890 | 0.766 | 367.5 |
| 100 | 100 | 0.2 | 0.715 | 0.659 | 0.762 | 356.9 |
| 1 | 200 | 0.7 | 2.072 | 2.143 | 0.866 | 786.1 |
| 10 | 200 | 0.5 | 1.584 | 1.648 | 0.723 | 774.1 |
| 50 | 200 | 0.4 | 1.127 | 1.131 | 0.779 | 751.6 |
| 100 | 200 | 0.3 | 0.917 | 0.890 | 0.766 | 735.1 |

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